**Unit 6 – Autonomous Differential Equations**

Goals/Rationale

By the end of this unit students will be familiar with phase lines and the classification of equilibrium solutions based on the long-term behavior of nearby solutions. The unit begins with students exploring the long-term outcome for a differential equation that models the population of owls, which turns out has both stable and unstable equilibrium solutions. The context of the problem will help them make sense of why this might be the case. They are also given the opportunity to invent their own one-dimensional representation of all solutions before formal introduction of the phase line. Phase line are introduced and students are given the opportunities to develop their own names for the different type of equilibrium solutions before the conventional names are given. In the last problem in the unit students are to develop both a dynamic sense of the phase line and a view of the phase line as a one-dimensional projection of all solutions.

**Page 6.1 – Analyzing autonomous DEs: Spotted Owls**

Implementation Notes

*Problems 1-2* – Do not hand out page 6.2 until this page is completed and no technology is to be used. The lack of technology will encourage students to think about the space of solution functions in terms of equilibrium solutions and the sign of the rate of change in between equilibrium solutions. Likely they will construct by hand a partial slope field, paying attention to whether the sign of dP/dt is zero, positive, or negative. Discussion of problem 1 might include the following:

* Why does it make sense for the population to die out if the initial population is less than 5 (scaled for hundreds)?
* What’s different about the equilibrium solution at P=8 and P=5?

In problem 2 students get the chance to essentially reinvent a phase line. Invite students to be creative and come up whatever one-dimensional representation of all solutions makes sense to them and to share their ideas with the class.

If it seems to fit, the instructor might ask students to do the following:

* Create your own differential equation that has exactly 2 repellors and 1 attractor.
* Is it possible to have a DE with exactly 2 constant function solutions both of which are attractors?

**Pages 6.2 – 6.4 – Phase lines**

*Problems 3-4* – Problem 3 introduces the term phase line and has students elaborate a partially completed phase line by adding arrows to indicate increasing or decreasing solutions. In problem 4 students come up with their own terms to describe stable and unstable equilibrium solutions. Be prepared for terms such as toilet bowl, firework, and other creative labels and why these labels make sense to them. Afterwards, the conventional terms of attractor and repellor should be introduced, as well as node. These are the terms used in the materials, but feel free to also introduce the terms stable, unstable, and semi-stable.

Problem 5 – This problem is fairly unique in that students are required to make explicit connections to movement or flow on the phase line to that of movement or flow of solutions on the P vs t graph. Parts (a) – (c) should be done and discussed before working on part (d). If available, one while and one red small unfix cube works well for part (c). If not, students can make their small objects from the corners of their papers. After students have thought about parts (a) – (c) on their own or with others, this is a fun problem to have one or more students come to the board or document cam and illustrate how their fingers move and why. If the following incorrect way of thinking for part (c) does not happen to come up, the following question makes for good discussion:

* Last semester when I taught this class someone argued that the little object on the phase line starting at P=1 always stays two units behind the little object that starts on the phase line at P=3. Do you agree or disagree with this idea and why?

In part (d) students should begin to see the phase line a one-dimensional projection of all solutions in the P vs. t plane. One way to do this is to imagine your eyes at the level of the piece of paper showing all the solutions in the P vs. t plane, then what you see is a one dimensional projection. All graphs of solutions in any particular region (e.g., between any two equilibrium solutions) have the exact same project. Encourage students to make connections to this fact and the fact that solutions to autonomous differential equations are shifts of each other along the t-axis.

**Notes for Personal Reflections on Unit 6**